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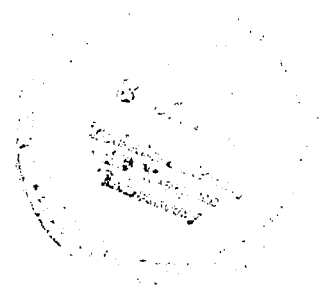
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# LIQUID-FILLED TRANSIENT PRESSURE MEASURING SYSTEMS: A METHOD FOR DETERMINING FREQUENCY RESPONSE

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16. Abstract  An equation is given and experimentally verified for computing the resonant frequency of liquid-filled transient pressure measuring systems. Resonant frequencies of 100 to 1000 Hz are typical of those systems tested. The effect of noncondensable gas bubbles on system response is described. A method for determining transducer volumetric compliance is presented. An example system is described and analyzed to demonstrate the use of the theory.					
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# LIQUID-FILLED TRANSIENT PRESSURE MEASURING SYSTEMS:

## A METHOD FOR DETERMINING FREQUENCY RESPONSE

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### SUMMARY

This report gives an equation for computing the resonant frequency of liquid-filled transient pressure measuring systems. The equation is based on a linearized second-order model and is verified by comparing the true measured frequency of several systems with the computed frequency. The measured frequency for each system tested fell within the expected error for that system. The resonant frequencies of the systems tested ranged from 100 to 1000 hertz.

The effect of gas bubbles on the response of liquid-filled systems is discussed. Gas bubbles act to lower the resonant frequency of liquid-filled systems. It is shown that gas bubbles can result from the so-called vacuum-fill method commonly used to fill liquid systems. In this method the system is evacuated before it is filled. An equation is presented for computing probable bubble effects from this fill method.

One of the more important numbers needed to compute the frequency response of liquid-filled systems is the compliance of the transducer. The compliance is due to the change in transducer volume caused by pressure changes.

A fairly simple method for determining transducer compliance is given in this report. This method is used on several commercial transducers, and the compliance values correlate with values of mechanical resonant frequency commonly given by manufacturers. The compliance measurement is sensitive to gas bubbles in the system. So a method is described to detect gas bubbles by measuring compliance at two different pressures.

An example system is described and analyzed to demonstrate use of the theory. The effect on the resonant frequency of gas bubbles and of a more compliant transducer is computed for the example system.

## INTRODUCTION

With the constant drive to better performance in aerospace devices, there is an increasing need to know the dynamic characteristics of these devices. One area of aerospace instrumentation strongly affected by this trend is pressure measurement. In this area, there is an increasing need to measure high-frequency pressure oscillations. Improvements in pressure transducers have helped to provide better transient response. In many cases, however, the transducer must be coupled to the source of pressure to be measured in such a way that the coupling limits the transient response of the resultant measuring system. Practical limits such as size and temperature often dictate that some length of tubing be used to transmit the pressure signal from the test apparatus to the transducer. The frequency response of the measuring system is then often limited by the response of this pressure transmission system.

The general problem of the frequency response of tube-and-volume pressure measuring systems has received considerable attention in the literature. Theoretical analyses based on linearized second-order systems (ref. 1) and on more rigorous models (refs. 2 and 3) are available. The special case where liquid is the fluid filling the system has been explored in references 1, 2, 4, and 5. The chief difference between the gas-filled and liquid-filled cases is that different parameters predominate in determining the resonant frequency. Often the primary parameter for the liquid-filled case is the transducer compliance, which is the change in volume of the transducer cavity caused by pressure changes. This parameter is not commonly specified by transducer manufacturers. Direct measurement of compliance for commonly used transducers is difficult because of the extremely small volume changes (typically  $10^{-13}$  to  $10^{-15}$  m<sup>3</sup>/(N/m<sup>2</sup>)) to be measured. Also, deviations from the ideal model are often encountered because of entrapped gas bubbles. Because of these bubbles and because compliance values are not generally available, the available theoretical analyses for predicting the frequency response of liquid-filled pressure measuring systems are not used as much as they could be.

A program was undertaken at Lewis to promote the capability of designing a liquid-filled pressure measuring system with predictable frequency response. The specific goals of this program were to

- (1) Compare available theoretical models for liquid-filled pressure measuring systems and obtain experimental verification of the predicted response
- (2) Devise a simple, reliable technique for measuring the compliance of commercial pressure transducers
- (3) Develop ways of including the effects of gas bubbles on the predicted response

This report presents the results of this work. It is intended to be a guide to experimenters and instrument system designers who wish to design liquid-filled pressure

measuring systems with predictable frequency response or to evaluate the frequency response of existing pressure measuring systems.

The particular problem considered in this work related to pressure measurements in alkali-metal heat-transfer loops. Transient pressure measurements are required in alkali-metal work in order to measure boiling and condensing instabilities and to measure instantaneous fluid velocities in heat exchangers so as to determine heat-transfer coefficients. Transient pressure measurements are also needed to determine dynamic characteristics necessary for control system design. The requirements for these transient pressure measurements included a frequency response up to about 100 hertz. Depending on the type of transducer used, the transducer operating temperature could range from 400 to 750 K. The transducer would be connected to the heat-transfer loop by tubing with inside diameters ranging from 0.003 to 0.009 meter. The minimum length of the tubing is that necessary to allow the transducer to be maintained at an acceptable operating temperature. Fluid temperature in the heat-transfer loops can range from 400 to 1500 K, depending on the location of the pressure measurement with respect to the components of the loop. Safety considerations require that all tube connections to the transducers be welded. Venting ports to allow trapped gases to be vented out of the transducers are not permitted.

The following text is divided into three sections. In the first section, the mathematical model for a liquid-filled pressure transducer is described, and the effects of uncertainties in the parameters are discussed. The second section describes the experimental verification of the mathematical model. The second section also includes a description of a technique for determining the compliance of a transducer and a method of checking to ensure that entrained gas bubbles have not affected the compliance measurement. The third section is a discussion of the application of these results to the problem of transient measurements in an alkali-metal loop. Also included in the third section is a discussion of the effect of entrapped gas in the pressure measuring system on normal filling procedures.

## PRELIMINARY SYSTEM ANALYSIS

### Mathematical Model

Several mathematical models of tube-and-volume pressure measuring systems are in the literature, as mentioned in the INTRODUCTION. From these models one of the less complex analyses, given in reference 1, has been chosen as a basis for the analysis in this report. This analysis assumes that the tube-and-volume system will respond as a linear second-order system.

There are, of course, more exact analyses in the literature. Iberall has solved, in reference 2, the fundamental flow equations to derive the equation for the dynamic response of a single tube-and-volume system. A more general and complex formulation, given in reference 3, extends the work of reference 2 and gives an equation for the dynamic response of a series-connected system of tubes and volumes. We have made comparisons between results from the reference 1 equation and results from the more exact equations (refs. 2 and 3). The results agreed to within 2 percent for the systems considered. Because the results were close and because the reference 1 equation is less complex, the linear-second-order model is used as a basis for the following development.

The development of the basic equation is given in reference 1 and is not repeated here. The equation from reference 1 gives the undamped natural frequency of a liquid-filled tube-and-volume system, with a square velocity profile in the tube assumed, as

$$f_n = \frac{1}{2\pi} \sqrt{\frac{\pi D^2}{4\rho LC}} \quad (1)$$

where

$f_n$  undamped natural frequency

$D$  tube inside diameter

$\rho$  liquid density

$L$  tube length

$C$  overall system compliance

(A complete symbol list is presented in appendix A.)

The equation for the fraction of critical damping in liquid-filled systems referred to as the damping ratio in reference 1 is

$$\xi = \frac{32\mu}{\pi D^3} \sqrt{\frac{\pi LC}{\rho}} \quad (2)$$

where  $\mu$  is the dynamic viscosity of the liquid. To use these formulas, the overall system compliance  $C$  must be expanded into definable, measurable quantities.

The overall system compliance for liquid-filled tube-and-volume systems is the sum of a number of terms, the three predominant ones being (1) the compliance of the transducer  $(\Delta V/\Delta P)_{TD}$ , (2) the compliance of the liquid in the transducer cavity  $V_{TD}/B$ , and (3) the compliance of the liquid in the tube  $4V_{TB}/\pi^2 B$ . This third term is less than the total compliance because the liquid compliance in the tube is considered distributed, not

lumped as in the first two terms. Various authors (refs. 1 and 6) have used one-half and one-third of the total compliance for the third term. We define this term such that the natural-frequency equation (eq. (3)) reduces to the closed-organ-pipe equation when the transducer volume  $V_{TD}$  is zero. With the expanded compliance term the equation for natural frequency may be written

$$f_n = \frac{a}{4L} \cdot \sqrt{\frac{V_{TB_e}}{B \left( \frac{\Delta V}{\Delta P} \right)_{TD} + V_{TD} + V_{TB_e}}} \quad (3)$$

where

$a$  liquid acoustic velocity

$V_{TB_e}$  effective tube volume

$B$  liquid bulk modulus

$\left( \frac{\Delta V}{\Delta P} \right)_{TD}$  transducer compliance

$V_{TD}$  transducer volume

In this form the relative influence of the parameters can easily be seen.

## Usable Frequency Range and System Damping

For this work the usable frequency range of a measuring system will be defined as the range from zero frequency to some upper frequency limit. This upper frequency limit is taken as the frequency where the amplitude ratio  $A = P_i/P_t$  just exceeds some chosen value. In general, the amplitude ratio as a function of frequency depends upon the system natural frequency and the damping in the system. However, the following argument is used to show that for water, and many other heat-transfer fluids, damping is low enough to neglect. The upper frequency limit then becomes a function of the natural frequency only.

For fluid-filled tubes in the presence of oscillating pressure the primary damping mechanism is the viscous interaction of the fluid with the tube walls. In references 2, 7, and 8 it is shown that the relative importance of this fluid-wall interaction is indicated by a dimensionless parameter called the Stokes number in reference 7. The equation is

$$S = \frac{\omega D^2}{\nu} \quad (4)$$

where

S Stokes number

$\omega$  angular frequency

D tube diameter

$\nu$  kinematic viscosity

In appendix B it is shown how the Stokes number at the natural (peak amplitude) frequency can be related mathematically to the fraction of critical damping parameter found in linear second-order system theory. The relation is given by

$$\xi = \frac{16}{S_n} \quad (B6)$$

where

$\xi$  fraction of critical damping

$S_n$  Stokes number at natural frequency

Because of the low kinematic viscosity of the liquids involved in this work, Stokes numbers at the natural frequency of typical systems ranged from 18 000 to 300 000. These Stokes numbers yield very low fractions of critical damping from  $8.9 \times 10^{-4}$  to  $5.4 \times 10^{-5}$ .

The formula for the fraction of critical damping used for these results has been shown to yield damping fractions which differ from experimental values by as much as 100 percent (ref. 5). However, even with these large errors, the fraction of critical damping is low enough to be negligible. Thus, the liquid-filled systems of interest in this work are considered undamped.

For undamped second-order systems, the upper limit of usable frequency can be calculated from

$$f_{\max} = f_n \sqrt{1 - \frac{1}{A_{\max}}} \quad (5)$$

where  $A_{\max}$  is the maximum tolerable amplitude ratio. For example, if the amplitude ratio is not to exceed 1.05, the upper limit of usable frequency  $f_{\max}$  must be equal to  $0.22 f_n$ .



## Error Analysis

Given the equation for computing natural frequency (eq. (3)), it is of interest to find out how sensitive the equation is to errors in the measured values used by the equation. An error analysis is especially important in this case because the computed natural frequency cannot easily be checked once the measuring system is installed. The user of the system must then rely solely on the theoretical value of the natural frequency to determine the usable frequency range.

To begin the error analysis, the frequency equation (eq. (3)) is written in a slightly different form

$$f_n = \frac{a}{4L} \sqrt{\frac{V_{TB_e}}{Z}} \quad (6)$$

where

$$Z = B \left( \frac{\Delta V}{\Delta P} \right)_{TD} + V_{TD} + V_{TB_e}$$

It is assumed that all errors are random. Then, when the well-known root-sum-square summation of errors is used, equation (6) yields

$$\begin{aligned} \frac{df_n}{f_n} = & \left( \left[ \frac{1}{2} \left[ 1 - \frac{B \left( \frac{\Delta V}{\Delta P} \right)_{TD}}{Z} \right] \right]^2 + \left( \frac{1}{2} \frac{d\rho}{\rho} \right)^2 + \left( \frac{1}{2} \frac{V_{TD}}{Z} \frac{dV_{TD}}{V_{TD}} \right)^2 \left[ \frac{1}{2} \left( 1 + \frac{EV_{TB}}{Z} \right) \frac{dL}{L} \right]^2 \right. \\ & \left. + \left[ \left( 1 - \frac{V_{TB_e}}{Z} \right) \frac{dD}{D} \right]^2 + \left[ \frac{1}{2} \frac{B \left( \frac{\Delta V}{\Delta P} \right)_{TD}}{Z} \frac{d \left( \frac{\Delta V}{\Delta P} \right)_{TD}}{\left( \frac{\Delta V}{\Delta P} \right)_{TD}} \right]^2 \right)^{1/2} \quad (7) \end{aligned}$$

## Gas Bubble Effects

Gas bubbles lower the resonant frequency of liquid-filled systems by increasing the volumetric compliance. The effect of a bubble on the calculated natural frequency may be found by adding the compliance of the bubble to the compliance of the transducer before using equation (3). To determine the bubble compliance, assume adiabatic

processes, and differentiate the equation of state; the result is

$$\left(\frac{\Delta V}{\Delta P}\right)_b = \frac{V_b}{\gamma P_b} \quad (8)$$

where

$V_b$  bubble volume

$\gamma$  specific-heat ratio for bubble gas

$P_b$  average bubble pressure

When the bubble compliance is added to equation (3), the equation is

$$f_n = \frac{a}{4L} \sqrt{\frac{V_{TB_e}}{B \left(\frac{\Delta V}{\Delta P}\right)_{TD} + B \left(\frac{V_b}{\gamma P_b}\right) + V_{TD} + V_{TB_e}}} \quad (9)$$

In equation (9) the relative size of the bubble term determines the effect of the bubble on the frequency value.

It is obvious that to achieve maximum frequency range, liquid-filled systems must be filled without noncondensable gas bubbles. In some cases a good fill has been achieved by using bleed holes to bleed off the gas after filling. But in other cases, especially liquid-metal systems, bleeding is not practical. In these cases, the vacuum-fill method is usually used.

In the vacuum-fill method the system is first evacuated. Then the system is filled with the working fluid. This procedure leaves noncondensable gas in the system, because the evacuation cannot be perfect. At various points in the system during filling the liquid will completely block the tubes leading to the pressure transducers. All the noncondensable gas left in the tube and volume at the that time will be compressed to bubbles in these transducers. The bubble volume may be calculated as a function of initial and final pressures and the transducer system volume, if the initial and final temperatures are assumed to be equal. From Boyle's law the expression for the final bubble volume is

$$V_b = \frac{P_1}{P_2} (V_{TD} + V_{TB}) \quad (10)$$

where

$V_b$  bubble volume  
 $P_1$  initial pressure  
 $P_2$  final pressure  
 $V_{TD} + V_{TB}$  measuring system volume

The bubble volume is known, so the equation for bubble compliance is

$$\left(\frac{\Delta V}{\Delta P}\right)_b = \frac{1}{\gamma} (V_{TD} + V_{TB}) \frac{P_1}{(P_2)^2} \quad (11)$$

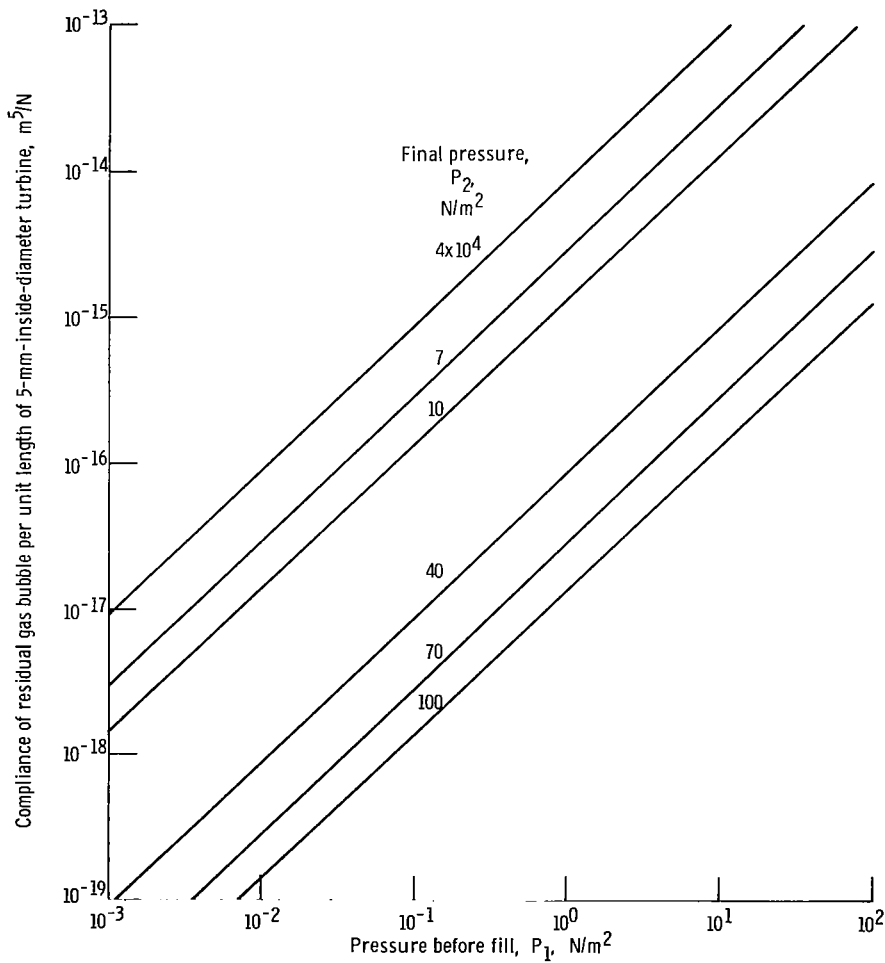


Figure 1. - Compliance of noncondensable gas bubble left in liquid-filled system after evacuation and fill as function of pressure to which measuring system was evacuated before it was filled. Specific heat ratio assumed equal to 1.4; initial and final temperatures assumed equal.

As a graphical aid for finding the bubble compliance in a system, bubble compliance per unit length of 5-millimeter-inside-diameter tubing is plotted as a function of initial pressure in figure 1. The transducer volume is assumed to be zero, and the initial and final temperatures are assumed to be equal. The value of the ratio of specific heats  $\gamma$  is assumed to be 1.4.

The compliance values in figure 1 can easily be corrected for different temperatures, volumes, tube diameters, or specific-heat ratios. For a temperature change, multiply the figure 1 value by the ratio of final temperature to initial temperature. For significant volumes, multiply the figure 1 value by the ratio of total measuring system volume to tube volume. For diameter change multiply the figure 1 value by the square of the ratio of new diameter to 5 millimeters. For different specific-heat ratios multiply the figure 1 compliance value by the ratio of 1.4 to the new specific-heat ratio.

## EXPERIMENTAL VERIFICATION

To verify the resonant frequency equation (eq. (3)) the natural frequency of a number of simulated measuring systems was measured. This measured frequency was then compared with the calculated frequency for each system. A summary of the data for these experiments is given in table I.

In every case the difference between the measured frequency and the calculated frequency was within the expected error. The expected error value was computed by using equation (7). The variable values and uncertainties used are given in table I. It may be concluded, then, that the frequency equation (eq. (3)) is adequate within the experimental error.

## Test System Description

The verification test systems were water-filled and had three main components, as shown in figure 2: (1) a cavity with a known compliance and volume, (2) a tube, and (3) a stiff pressure sensor.

Cavity. - The "cavity" in these simulated measuring systems was a stainless-steel bellows with a volume of  $1.52 \times 10^{-3}$  cubic meters and a compliance of  $2.7 \times 10^{-15}$  meter<sup>5</sup> per newton. The compliance of the bellows was determined by first completely filling the inside of the bellows and then changing the  $\Delta P$  across the wall. The pressure transmitting tube was attached to a capillary tube, so the change in internal volume registered as liquid rise in the calibrated capillary tube. The stated value is an average over 10 readings and has an uncertainty of 10 percent.

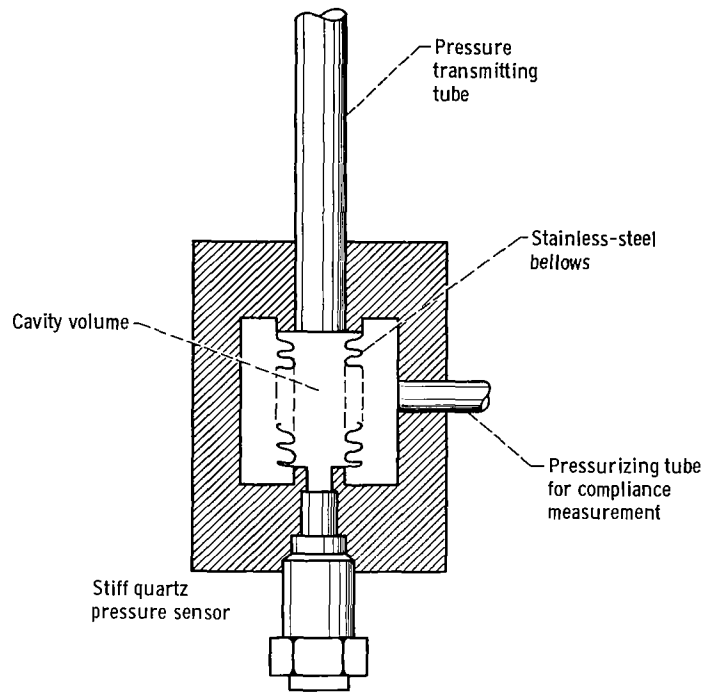


Figure 2. - Diagram of simulated measuring system.

The bellows volume was determined by weighing the bellows, first empty and then completely filled with water. The weight difference was divided by the density of water to compute the volume.

Tubes. - The tubes for these simulated measuring systems were copper or stainless steel and had inside diameters ranging from  $0.31 \times 10^{-2}$  to  $0.78 \times 10^{-2}$  meter and lengths from 0.132 to 0.759 meter. Both ends of each tube were carefully deburred to eliminate any area changes caused by the cutting operation.

Pressure sensor. - The stiff pressure sensor in these systems was a miniature commercial piezoelectric pressure transducer with a compliance less than  $2.38 \times 10^{-17}$  meter<sup>5</sup> per newton. This value was computed by using the manufacturer's values for diaphragm displacement and effective area. This device was chosen because its small compliance would make a negligible contribution to the dynamics of the systems.

## Filling of Test Systems

The experimental systems were filled by using the apparatus diagrammed in figure 3. The filling equipment included a reservoir filled with tap water and a length of hypoder-

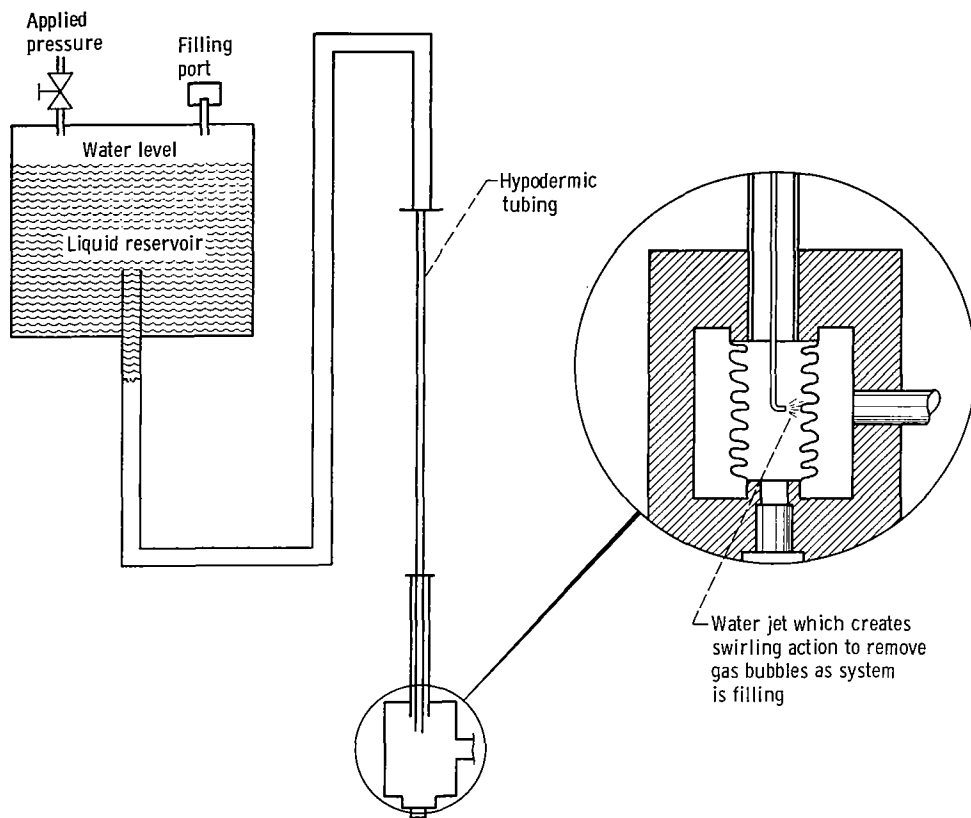


Figure 3. - Apparatus for filling tube-and-volume systems.

mic tube with a hooked end. The hooked hypodermic tube was inserted into the test system and the reservoir was pressurized to force water in. The water jet from the hooked end created a swirling action in the cavity that washed bubbles out of the test system. No attempt was made to remove dissolved gases from the water. Therefore, to reduce the possibility of gas bubble formation before tests began, the tests were run very soon (at most 2 min) after the system was filled. Also, the reservoir and the test systems were kept at constant temperature.

## System Excitation

The natural frequency of a system may be determined by observing the output when applying either a step, impulse, or sinusoidal input. For this work, both the sinusoidal and impulse inputs were used. For sinusoidal input, the frequency was swept from 10 to 1000 hertz. The resonant frequency was defined as the frequency at which there was a  $90^\circ$  phase lag between input and output pressure signals. For impulse input, the resonant

frequency was defined as the frequency of oscillations occurring during the short time after the impulse was applied.

The sinusoidal pressure input was produced by applying sinusoidal acceleration to a column of water. The mouth of the test system was placed 10 centimeters below the surface of a container of water attached to an electrodynamic shaker (see fig. 4). Variable-frequency sinusoidal acceleration was applied to the liquid column through the shaker system. The pressure input to the test system was measured by a pressure transducer mounted in the side of the container at the 10-centimeter depth.

The frequency of the input pressure oscillations was variable from 10 to 1000 hertz. The root-mean-square input amplitude at low frequencies was limited to  $1.4 \times 10^3$  newtons per square meter because higher amplitudes caused sloshing which distorted the pressure waveform. The root-mean-square input amplitude at higher frequencies was less than  $1.0 \times 10^3$  newtons per square meter because of shaker acceleration limits. The usable frequency spectrum of the pressure generator above 700 hertz was not continuous, because resonance of the container and other parts of the system caused waveform distortion which precluded measurements at certain frequencies.

The impulse pressure input was produced by striking the experimental system in a direction parallel to the centerline of the test system tube. The output (examples shown

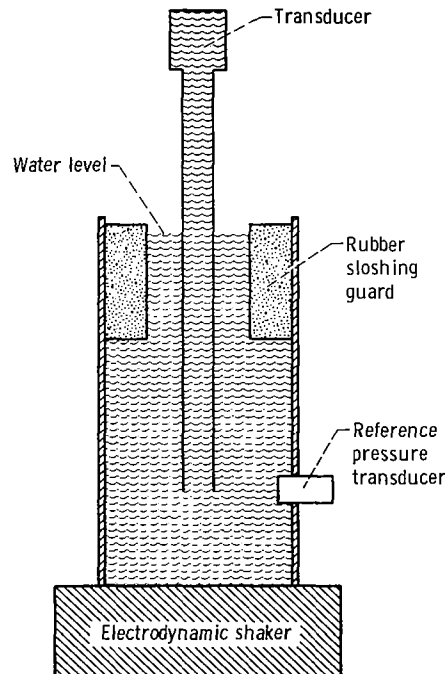
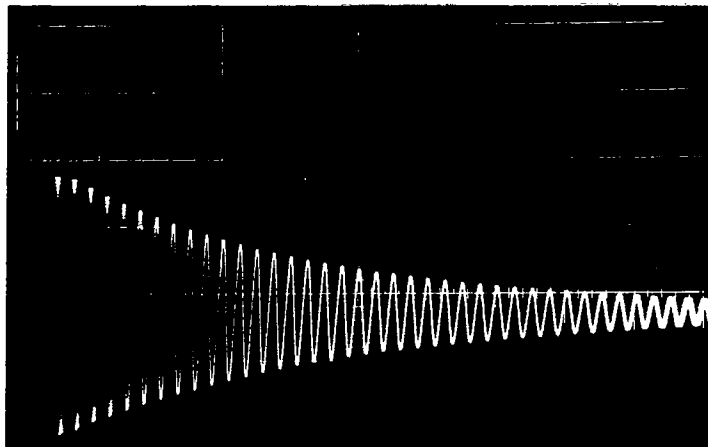
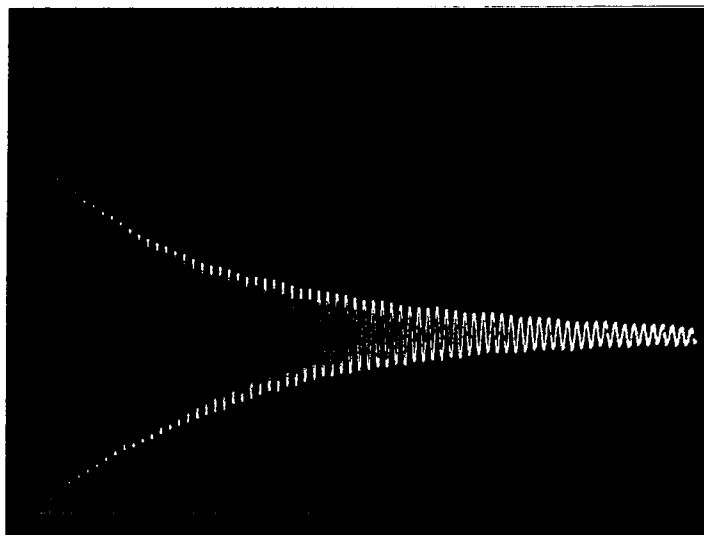


Figure 4. - Sinusoidal liquid pressure generator.



(a) Transducer, MB Model 151; transducer range, 0 to  $340 \times 10^3$  newtons per square meter; frequency, 200 hertz; tube length, 0.129 meter; tube inside diameter,  $0.457 \times 10^{-2}$  meter; transducer volume,  $0.234 \times 10^{-5}$  cubic meter; compliance,  $79 \times 10^{-15}$  meter<sup>5</sup> per newton.



(b) Transducer, Data Sensor model PB923A; transducer range, 0 to  $1700 \times 10^3$  newtons per square meter; frequency, 360 hertz; tube length, 0.305 meter; tube inside diameter,  $0.457 \times 10^{-2}$  meter; transducer volume,  $0.112 \times 10^{-5}$  cubic meter; compliance,  $8.8 \times 10^{-15}$  meter<sup>5</sup> per newton.

Figure 5. - Typical output traces from impulse excitation method. Sweep speed, 20 milliseconds per centimeter; liquid, tap water.



in fig. 5) was photographed from an oscilloscope trace. The photos were then analyzed for values of natural frequency.

This impulse technique is more convenient to use. Only an oscilloscope, camera, and the electronics necessary to display the transducer signal on the scope are needed. The errors in the measurement are controllable. Frequency value uncertainties can only result from uncertainties in the distance measurement on the photograph and uncertainties in the sweep speed.

One additional advantage of this impulse method is that it offers a simple way to detect bubbles in the system. Since the compliance of a bubble is inversely proportional to pressure, the presence of a bubble can be detected by measuring the natural frequency at two different pressure levels. Constant natural frequency with pressure means there are no bubbles in the system large enough to affect the dynamic response of the system at those pressures.

The impulse method was used in testing all of the commercial transducer systems mentioned in the next section because of its convenience.

## Determining Transducer Compliance

A value for transducer compliance is needed to adequately predict the natural frequency for liquid-filled systems. Direct measurement of compliance is difficult, and transducer manufacturers rarely include compliance values in their specifications. One way of finding the compliance is to solve the natural-frequency equation for  $(\Delta V/\Delta P)_{TD}$ . The compliance can then be calculated from given values for natural frequency, cavity volume, tube length and diameter, and liquid parameters. Of course, this means that the accuracy of the compliance value depends on the adequacy of the natural-frequency equation. But as was stated previously, the equation yields results within the expected error for the geometries used in this work. Solving equation (3) for compliance yields

$$\left(\frac{\Delta V}{\Delta P}\right)_{TD} = \frac{V_{TB_e}}{B} \left[ \left( \frac{\frac{a}{4L}}{f} \right)^2 - 1 \right] - \frac{V_{TD}}{B} \quad (12)$$

This method of computing compliance was used on several commercially available transducers by using the impulse method to measure the natural frequency. The compliance values correlated fairly well with the mechanical natural frequency given by manufacturers.

In figure 6, the compliances for 16 commercial transducers are plotted against their specified mechanical natural frequencies. Figure 6 may be used to estimate compliance

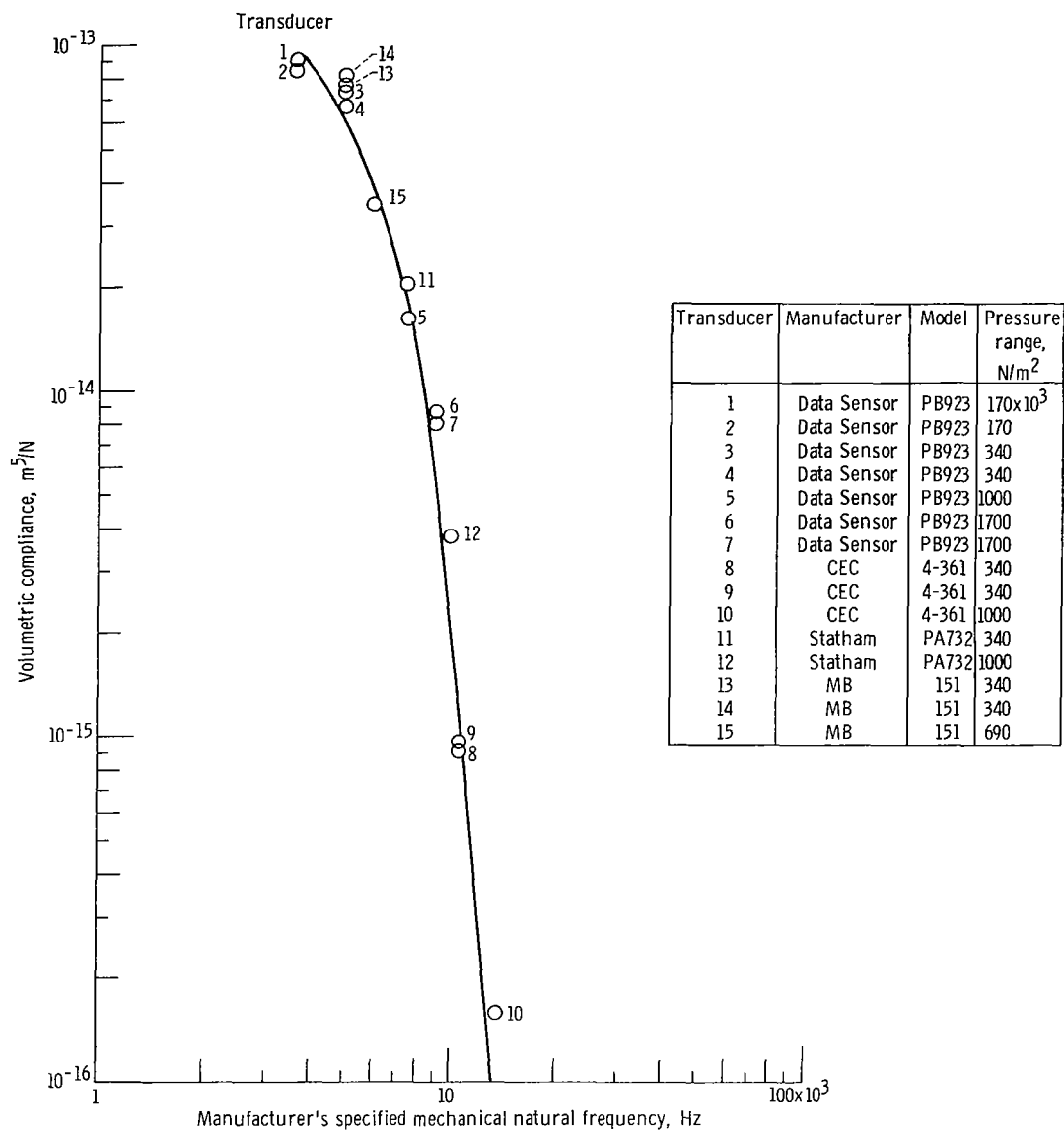


Figure 6. - Volumetric compliance as function of mechanical resonant frequency for metal-strain-gage-type absolute-pressure transducers.

for metal-strain-gage-type pressure transducers whose resonant frequencies fall within the range covered by these tests (3 to 11 kHz). It is felt that figure 6 is a valid tool because the population tested includes (1) five pressure ranges (from  $170 \times 10^3$  to  $1700 \times 10^3$  N/m<sup>2</sup>), (2) several internal volume sizes (from  $0.98 \times 10^{-6}$  to  $1.3 \times 10^{-6}$  m<sup>3</sup>), (3) several tube volumes (from  $1.4 \times 10^{-6}$  to  $8.6 \times 10^{-6}$  m<sup>3</sup>), and (4) two types of strain-gage designs (bonded and unbonded). Figure 6 does not include data for miniature strain-gage transducers (diameter less than 1 cm) or for semiconductor-strain-gage-type transducers.

It should be noted that figure 6 probably cannot be used for non-strain-gage transducers or for strain-gage transducers which vary markedly from the transducers tested.

## APPLICATIONS

When frequency response information is needed for a system and the frequency response of the installed system cannot be measured, the user will normally try to compute the natural frequency of his system, from which he can derive the system response. This section describes a typical system and the computations for this system. The discussion shows how noncondensable gas bubbles influence the computed frequency response. In addition, the importance of the transducer compliance is emphasized by showing what happens to the results when the compliance is altered by changing transducers.

### Example System Description

The example pressure measuring system is to be installed in an alkali-metal heat-transfer loop. The geometry of the pressure measuring system in such a loop greatly depends on the operating temperature. Higher temperatures require that the pressure transducer be extended out from the main stream in order to allow the transducer to cool below its high temperature limit. This standoff tube should be short to keep the resonant frequency high if a wide usable frequency range is needed. So the optimum transducer for this case is one which can operate at high temperatures and thus allow use of short standoff tubes.

A high-temperature unbonded strain-gage transducer is selected for the example system. This transducer operates reliably at 530 K (500<sup>o</sup> F). The volume was measured as 1.64 cubic centimeters, and the compliance was measured as  $0.16 \times 10^{-15}$  meter<sup>5</sup> per newton. The range of the transducer is  $1030 \times 10^3$  newtons per square meter. Liquid potassium at 533 K is used to fill the system. The system geometry and the thermal properties of the liquid are summarized in table II.

### Example System Analysis

Frequency response. - The natural frequency of the sample system is 1010 hertz when equation (3) and data from table II are used.

Given this natural frequency, the usable frequency range of the sample system is from 0 to 220 hertz.

Effect of changing transducer. - To show how changes in the sample system can grossly affect the system response, a second transducer is selected for comparison. The system with this second transducer is called system B. System A is the system with the original transducer. This new transducer has a smaller volume ( $V_B = 0.22 V_A$ ) and a larger compliance ( $\Delta V/\Delta P_B = 24 \Delta V/\Delta P_A$ ).

The natural frequency of system B is 615 hertz compared with the 1010 hertz of system A. This is a 40-percent reduction in usable frequency range.

Bubble effects. - With either system A or system B the computed system response is accurate within the random error uncertainty when the system is assumed to be completely filled. In reality the system probably will not be filled completely and will contain some undissolved gas bubbles. Especially with liquid-metal systems, where gas bleeding is not commonly used, the evacuation method of filling described in the section Gas Bubble Effects leaves gas in the system.

Computations were made by using equations (8) and (9) to show how various pressure values during filling can affect the natural frequency. Results are given in table III for both system A and system B. The temperature is assumed constant, and the entire measuring system volume is assumed to be the initial volume for the bubble computations. Also, the specific-heat ratio is assumed to be 1.6. Two conclusions may be drawn from table III. First, it is clear that the bubble effects can be reduced if the initial pressure is low enough. Second, it is apparent that system A, with a higher resonant frequency, is more susceptible to bubble effects on a percentage basis.

## CONCLUDING REMARKS

A linearized second-order model of tube-and-volume liquid-filled pressure measuring systems has been verified as a valid predictor of the resonance frequency of systems whose geometry lies within the geometry range studied in this work. Other more complex theories were examined, but they did not improve the results enough to justify the extra computing they require.

The resonant frequency was measured for several experimental systems to verify the linearized second-order model. The measured natural frequencies were close to the computed frequencies within the expected error for the systems tested. Error analysis of the theory gave from 2.5 to 4.6 percent expected random error for the computed frequencies.

To use the theory on a given system, measurements of system parameters must be made. One important parameter predominant in liquid-filled system analysis is the transducer compliance, or the change in volume of the transducer with pressure, caused by flexing of the transducer diaphragm. A fairly simple, reliable method for finding

compliance values was presented in this report. Compliance values for some commonly used strain-gage-type absolute-pressure transducers were determined by this method. These compliance values correlated with manufacturers' given mechanical resonance values. The graph of these values may be used to estimate the compliance of transducers similar in type and size to those transducers tested.

One of the practical problems in filling these systems is gas bubbles which remain after filling. A procedure for detecting gas bubbles in a liquid-filled system by using the compliance measurement method was devised and used for the experiments in this work. Bubbles were detected by determining compliance at two different pressures.

For cases where the natural frequency cannot be measured and the vacuum-fill technique must be used, an equation was given for computing bubble compliance. The bubble effects on the system response can easily be computed by adding the compliances of the bubble and the transducer in all computations.

An example system was described and analyzed to demonstrate the use of the theory and to emphasize some of the important points. To show the importance of the transducer compliance, a more compliant transducer was put into the example system. Bubble effects in the example were studied by assuming the vacuum-fill method was used. It was shown that, if the initial pressure is not low enough, the resulting bubble could cause a large decrease in the resonant frequency from what it would be for the completely filled system.

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National Aeronautics and Space Administration,  
Cleveland, Ohio, October 6, 1971,  
112-27.

## APPENDIX A

### SYMBOLS

A	amplitude ratio
a	acoustic velocity, m/sec
B	liquid bulk modulus, $\text{m}^2/\text{N}$
C	overall system compliance, $\text{m}^5/\text{N}$
D	tube inside diameter, m
f	frequency, Hz
L	tube length, m
P	pressure, $\text{N}/\text{m}^2$
S	Stokes number
V	volume, $\text{m}^3$
$\frac{\Delta V}{\Delta P}$	volumetric compliance, $\text{m}^5/\text{N}$
Z	denominator of eq. (6), $\text{m}^3$
$\gamma$	ratio of specific heats
$\mu$	dynamic viscosity, $\text{kg}/(\text{m})(\text{sec})$
$\nu$	kinematic viscosity, $\text{m}^2/\text{sec}$
$\xi$	fraction of critical damping
$\rho$	density, $\text{kg}/\text{m}^3$
$\omega$	angular frequency, rad/sec

#### Subscripts:

b	gas bubbles
e	effective
i	indicated by measuring system
n	natural frequency of system
TB	transmitting tube

TD transducer  
t true value being measured  
1 initial value (see fig. 1)  
2 final value (see fig. 1)

## APPENDIX B

### STOKES NUMBER AT RESONANT FREQUENCY RELATED TO FRACTION OF CRITICAL DAMPING

The equation for the fraction of critical damping in a liquid-filled system from reference 1 may be written

$$\xi = \frac{32\mu}{\pi D^3} \sqrt{\frac{\pi LC}{\rho}} \quad (B1)$$

Again from reference 1 the equation for resonant frequency is

$$\omega = 2\pi f = \sqrt{\frac{\pi D^2}{4\rho LC}} \quad (B2)$$

From reference 7 the equation for Stokes number is

$$S = \frac{\omega D^2}{\nu} \quad (B3)$$

where

$$\nu = \frac{\mu}{\rho}$$

Substituting equation (B2) for resonant frequency into the Stokes number equation (B3) yields the Stokes number at the natural frequency:

$$S_n = \frac{D^2 \rho}{\mu} \sqrt{\frac{\pi D^2}{4\rho LC}} \quad (B4)$$



Solving equation (B4) for  $\mu$  yields

$$\mu = \frac{D^2 \rho}{S_n} \sqrt{\frac{\pi D^2}{4 \rho L C}} \quad (B5)$$

Substituting equation (B5) into equation (B1) yields the fraction of critical damping as a function of the Stokes number at the natural frequency:

$$\xi = \frac{16}{S_n} \quad (B6)$$

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TABLE I. - DATA FROM VERIFICATION EXPERIMENTS

[Liquid, tap water; water temperature, 295 K; water bulk modulus<sup>a</sup>,  $(2.2 \pm 0.1) \times 10^9$  N/m<sup>2</sup>; water density<sup>a</sup>,  $(0.997 \pm 0.000) \times 10^3$  kg/m<sup>3</sup>; volume of simulated transducer,  $(1.52 \pm 0.05) \times 10^{-6}$  m<sup>3</sup>;  $\Delta V/\Delta P$  of simulated transducer,  $(2.7 \pm 0.3) \times 10^{-15}$  m<sup>5</sup>/N.]

Test	Tube inside diameter <sup>b</sup> , m	Tube length, m	Measured natural frequency, Hz	Calculated natural frequency, Hz	Deviation of calculated from measured frequency, percent (d)	Expected root-sum-square deviation, percent
1	$0.310 \times 10^{-2}$	$18.3 \times 10^{-2}$	529	533	0.8	4.6
2	.310	28.4	418	420	.5	4.5
3	.310	38.7	352	353	.3	4.3
4	.310	47.3	314	315	.3	4.2
5	.310	53.8	284	293	3.2	4.1
6	.475	13.2	962	940	-2.3	4.3
7	.475	17.0	808	817	1.1	4.2
8	.475	20.1	759	740	-2.5	4.1
9	.475	27.1	621	623	.3	4.0
10	.475	37.3	510	510	0	3.8
11	.475	38.6	501	499	-.4	3.7
12	.475	44.8	470	453	-3.6	3.6
13	.475	47.5	435	436	.2	3.5
14	.475	51.6	425	413	-2.8	3.5
15	.475	57.5	384	383	-.3	3.3
16	.475	69.5	335	337	.6	3.3
17	.475	73.3	332	325	-2.1	3.2
18	.617	24.4	807	810	.4	3.7
19	.617	34.6	644	641	-.5	3.3
20	.617	45.1	538	533	-.9	3.2
21	.617	55.2	463	461	-.4	3.0
22	.617	65.3	408	407	-.2	2.8
23	.617	75.6	364	363	-.3	2.8
24	.777	35.2	722	724	.3	3.0
25	.777	55.6	509	510	.2	2.6
26	.777	65.7	444	446	.5	2.5
27	.777	75.9	394	396	.5	2.5

<sup>a</sup>From ref. 9.

<sup>b</sup>Uncertainty,  $\pm 0.005 \times 10^{-2}$  m.

<sup>c</sup>Uncertainty,  $\pm 0.15 \times 10^{-2}$  m.

<sup>d</sup>Percent deviation =  $\frac{f_{\text{calc}} - f_{\text{meas}}}{f_{\text{meas}}} \times 100$ .

TABLE II. - DATA FOR EXAMPLE SYSTEMS

## (a) System geometry

Tube length, m . . . . .	0.305
Tube inside diameter, m . . . . .	$0.475 \times 10^{-2}$
Transducer volume for system A, $m^3$ . . . . .	$1.64 \times 10^{-6}$
Transducer volume for system B, $m^3$ . . . . .	$0.361 \times 10^{-6}$
Transducer compliance for system A, $m^5/N$ . . . . .	$0.16 \times 10^{-15}$
Transducer compliance for system B, $m^5/N$ . . . . .	$3.8 \times 10^{-15}$

## (b) Thermal properties of potassium

Temperature <sup>a</sup> , K . . . . .	533
Density <sup>b</sup> , $kg/m^3$ . . . . .	$0.78 \times 10^3$
Dynamic viscosity <sup>b</sup> , $kg/(sec)(m)$ . . . . .	0.279
Bulk modulus <sup>c</sup> , $N/m^2$ . . . . .	$2.31 \times 10^9$

<sup>a</sup>Temperature is assumed constant over tube length.<sup>b</sup>See ref. 10.<sup>c</sup>See ref. 11; value is derived from acoustic velocity and density values.

TABLE III. - RESULTS FOR COMPUTATION OF UNDISSOLVED GAS

## BUBBLE EFFECTS FOR VARIOUS PRESSURE VALUES

## DURING FILLING PROCESS

[Specific heat ratio  $\gamma$  assumed to be 1.6; initial and final temperatures assumed equal.]

Pressure before fill, $P_1$ , $N/m^2$	Pressure after fill, $P_2$ , $N/m^2$	System A (a)	System B (b)
		Normalized natural frequency	
1.3	$7 \times 10^5$	1.0	1.0
1.3	$7 \times 10^4$	.78	.90
13	$7 \times 10^5$	.97	.98
13	$7 \times 10^4$	.37	.55

<sup>a</sup>Based on a frequency of 1010 Hz.<sup>b</sup>Based on a frequency of 615 Hz.